ECONOMICS 311: Intermediate Microeconomic Theory
ASSIGNMENT 5

Answers to this assignment are due back by Monday, February 29, 2016.

1. **Graphing isoquants**

   The weekly production of computers is given by the following Cobb-Douglas function:
   \[ q = 100 K^{0.5} L^{0.5} \]
   a. Graph the isoquant for 10,000 computers (per week).
   b. Calculate the rate of technical substitution when
      a. \( K = 49 \)
      b. \( K = 100 \)
      c. \( K = 169 \).
      [Hint: calculate the value of \( K \) and \( L \) at these points; then raise \( K \) by 1 (so \( \Delta K = 1 \)) and calculate the new \( L \), which will allow you to get \( \Delta L \). A spreadsheet might help.] Assume \( q=10,000 \).
   c. For the case where \( K = 100 \), find the average productivity of labor, again assuming \( q=10,000 \).
   d. Show that this production function displays constant returns to scale.
   e. Technical change now makes production more productive, so the production function now becomes \( q = 120 K^{0.5} L^{0.5} \). Calculate the MRS for the point where \( K=100 \) (for the \( q=10,000 \) isoquant). How, if at all, does it differ from your answer to 1.b.b?
   f. For this new production function, find the average productivity of labor (for \( K = 100 \)).
   g. Labor, and capital services, each cost $10,000. Assuming \( L = K = 100 \), calculate the average cost of producing one computer. Here we have returned to the original production function.
   h. Keeping \( K = 100 \), compute the average cost of producing each computer for \( L = 50, L = 80, L = 100, L = 150 \), and \( L = 200 \). Graph these numbers. [Hint: Use Excel!]
   i. Use the numbers in h to compute and graph marginal cost.

2. **More isoquants**

   The output of mugs is given by the production function
   \[ q = 6 K + 3 L \]
   where \( K \) is the amount of capital and \( L \) the amount of labor, per day.
   a. Graph the \( q=150, q = 300, \) and \( q = 450 \) isoquants.
   b. Does this production function display diminishing marginal of labor? of capital? Explain.
   c. Does this production function show decreasing, constant, or increasing returns to scale? Explain.
   d. If the wage is $1 and the rental rate on capital is $1, what cost-minimizing combination of \( L \) and \( K \) should you use to produce 300 mugs? Explain.
   e. If the wage is $1, and the rental rate on capital is $5, what cost-minimizing combination of \( L \) and \( K \) should you use to produce 300 mugs? Explain.
3. **Metro Fares**

An urban rapid-transit line runs crowded trains (with 200 passengers per car) at rush hours, but nearly empty trains (20 passengers per car) at off-peak hours. A management consultant argues the following:

"The cost of running a car for one trip on this line is about $100, regardless of the number of passengers. So the per-passenger cost is about $0.50 at rush hour but rises to $5 per passenger in off-peak hours. Consequently, we had better discourage off-hour business."

a. Is the consultant’s argument correct? Explain.
b. Many metro systems sell monthly “commuter” tickets that allow for multiple rides at a reduced price per ride; these are mostly used by rush-hour riders. Are such tickets a good idea? Explain.

4. **Mechanics of cost curves**

A price-taking firm has marginal cost $MC = 20 + 10q$, average variable cost $AVC = 20 + 5q$, and fixed costs of 500, where $q$ refers to the output of the firm.

a. Write down the firm’s total cost ($TC$) function.
b. What is the firm’s profit-maximizing output if the market price is 100?
c. Is the firm making (economic) profit?

5. **Short questions**

a. A firm has increasing returns to scale and is making profits. The government is concerned that the firm has grown too big, and splits it into two smaller firms of equal size. What effect would this have on overall profits?
b. If $p MP_1 < w_1$, should the firm increase or decrease the amount of factor 1, assuming the firm wants to maximize its profits?